3501. Consider e.g. $y = x + 2$.

3502. Both top and bottom are zero at $x = 0.2$. Hence, $(5x - 1)$ must be a factor of both.

3503. Assume that each region is equally likely to be shaded or unshaded. Then there are 16 equally likely outcomes in the possibility space. Classify the successful ones by number of squares shaded.

3504. This is false: find a counterexample.

3505. These are quadratics, so use the discriminant.

3506. Show that $\frac{dy}{dx}$ is never equal to 1.

3507. (a) Set the cubic and the quadratic equal to each other, and solve for t.
(b) Find the first and second derivatives.
(c) Calculate x at $t = T + 10$ in each case.

3508. Find a counterexample involving parallel (x, y, z) planes. These are constructed in the same way as parallel (x, y) lines.

3509. (a) Draw a sketch and use Pythagoras.
(b) Call the ratio z and set $\frac{dz}{dk} = 0$. Use the second derivative to show that your SP is a minimum.

3510. Use a log rule to simplify. Then use the fact that the ln function is one-to-one and thus invertible over $(0, \infty)$ to show that f is one-to-one and thus invertible. To find the instruction of f^{-1}, set the output to y, and rearrange for x.

3511. The LHS is an infinite geometric series.

3512. (a) Use $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$.
(b) Find the probability that none occur.

3513. This is integration by inspection, using the reverse chain rule.

3514. For all parts, use the information concerning the value of the function f and its derivatives to find A and B in terms of k. Show that the curve $y = f(x)$ is an enlargement of $y = \sin x$ by scale factor $\frac{1}{k}$ (in both x and y).

3515. Set up and solve the equation

$$\frac{x}{2x-3} = \frac{2x+3}{x}.$$

Each side of this equation is equal to r^2, where r is the common ratio. So, produce an equation of the form $r^2 = a \pm \sqrt{b}$, and show that both values on the RHS are positive.

3516. Differentiate $f^{(k)}(x) = (x+k)e^x$ with respect to x, using the product rule.

3517. If M is large enough, the system will accelerate to the right, with friction at F_{max} acting up the slope. Set up an equation of motion along the string.

3518. Consider the transformation in the y direction as a replacement of the variable y by the variable $y - 2$.

3519. You don't need to sketch the hexagon. Consider it as six equilateral triangles. Use the sine area formula and multiply by six.

3520. (a) Consider the signs of $x + y$ and $x - y$ if $x \le 0$.
(b) Use a log rule and eliminate y^2 to produce a quadratic in x. Use the quadratic formula.

3521. One of these is necessarily true.

3522. Reflect the given parabola in the x axis.

3523. Solve the inequality $X^2 > kX$ first. Then consider the symmetry of the normal distribution.

3524. Find an expression for the range in terms of the angle of projection θ. Then use a double-angle formula.

3525. Only one of these is true.

3526. Draw in the square connecting the points on the circumference, and consider the formula $A = \frac{1}{2}bh$ for a triangle.

3527. Use the product rule, then $y - y_1 = m(x - x_1)$.

3528. Without loss of generality, place the vectors on the axes, such that $\mathbf{a} = a\mathbf{i}$ and $\mathbf{b} = b\mathbf{j}$. Then consider gradients.

3529. (a) Multiply three probabilities.
(b) Multiply your answer in (a) by the number of orders of RGB.

3530. Substitute for y. For $k \neq 0$, you'll get a quadratic in x. Use the discriminant.

3531. Perform the substitution, and then use partial fraction to find the relevant u integral.

3532. Set up a single definite integral, and integrate by parts. Let $u = 1 - \ln x$ and $v' = x$.

3533. (a) Show that $x = f(x)$ has no roots.
(b) Use the discriminant.

3534. The exponential factor doesn't affect things, as it is always positive. So, solve the relevant quadratic inequality.

3535. (a) Write $\tan x \equiv \frac{\sin x}{\cos x}$.
(b) Integrate by inspection with a standard result.

3536. This is true if f has no discontinuities. But there are counterexamples with vertical asymptotes.

3537. (a) Let p be the probability that fungus is found in any square metre of forest floor.
(b) Find, assuming H_0, the probability that the number of locations showing evidence of the fungus is greater than or equal to 19. Then compare this value to 0.5%.

3538. Write the binomial coefficients in factorials, and solve. Consider carefully whether any roots found do satisfy the original equation.

3539. (a) Include four forces on your diagram.
(b) Resolve parallel/perpendicular to the ladder.
(c) Assume limiting friction, i.e. $F_{\text{wall}} = \mu R_{\text{wall}}$.

3540. This is most easily done by solving the equation to find an explicit expression for x_n.

3541. This is best sketched by making x the subject.

3542. (a) The graph of an inverse function is a reflection in the line $y = x$.
(b) Use the reflection in (a).
(c) Carry out the integral in (b).

3543. Consider the side lengths of a triangle formed of two opposite (along a space diagonal) vertices of a cube, and another vertex. Such a triangle is right-angled, and contains the relevant angle.

3544. (a) Integrate NII twice, calculating the constant of integration each time.
(b) Find the range of the velocity, by consider the range of sine/cosine.

3545. You need to list the six ways, using a method of classification that guarantees you have all of them. Consider the cases in which the central square is shaded/not shaded.

3546. Firstly, calculate the area of the region in common to both circles, using segments. Then subtract this from the area of one circle.

3547. Write $a \equiv e^{\ln a}$, and use the reverse chain rule.

3548. Set up an equation for intersections, and factorise it using a polynomial solver.

3549. This is a quadratic in $\sin\theta$.

3550. (a) Consider the fact that the tension is the same throughout a light string passed over smooth objects.
(b) Draw a force diagram for the load, and resolve vertically and horizontally. The components of the contact force are reaction and friction.

3551. Firstly, take out a factor of x. The solution $x = 0$ is a straight line, so it gives no points of inflection. Then you can rearrange to make y the subject, and differentiate twice.

3552. Write down the mean μ, using the symmetry of a normal distribution. Then set up an equation for σ using an inverse normal for $1/4$ (or $3/4$).

3553. (a) Take out a factor of $\sqrt{9}$ and use the generalised binomial expansion.
(b) Solve for intersections by substituting for y and using the quadratic formula. Approximate the discriminant using your answer from (a).

3554. (a) This is a plan view, so the lines of action of weight and reactions are directly into the page.
(b) The other two equations you need are moments around the y axis and vertical (into the page) equilibrium.

3555. Draw a clear sketch, and use the fact that tangent is perpendicular to radius.

3556. Multiply together three binomial coefficients.

3557. Set up the first principles formula. Eliminate the inlaid fractions. Then multiply top and bottom of the result by the conjugate of the numerator, i.e. by $\sqrt{x} + \sqrt{x+h}$. This will allow you to cancel a factor of h on the top and bottom, and so take the limit.

3558. Find the equation of the tangent.

3559. Eliminate the logs from the equation, so that you can sketch the family of curves. The end result, however, will be a shaded region, not a discrete set of curves.

3560. The implication goes forwards only.

3561. Write the sum longhand, and then multiply up by the denominators of the fractions.

3562. Show first that $y = \cot x$ has a point of inflection, then consider reflections in $y = x$.

3563. Integrate both equations with respect to t. Then substitute the second equation into the first. At the end, use the given point to find the constants of integration.

3564. Multiply up by $x^3 - 1$, and equate coefficients. Adding all three resulting equations together will give you C.

3565. (a) Consider the prime factors of P.
(b) Compare sizes.
(c) Find the contradiction and finish the proof.

3566. Sketch the path. Then use the exact trig ratio $\tan 30° = \sqrt{3}/3$ to calculate lengths/coordinates.

3567. Set up horizontal and vertical *suvat*s. Eliminate t to find the Cartesian equation of the trajectory.

3568. Substitute $-x$ for x, and show that, the x value is negated, so is the y value.

3569. Find the probability that the first three cards dealt are the same suit, and the last is different. Then multiply this by the number of orders of SSSD.

3570. Use the fact that a cubic has rotational symmetry around its point of inflection, choosing p to be the x coordinate of the point of inflection.

3571. Factorise the LHS in each proposed implication, and consider the range of the sine function.

3572. Take out a factor of $x^{\frac{1}{5}}$, and you are left with a quadratic in $x^{\frac{1}{5}}$.

3573. (a) The Cartesian unit circle $x^2 + y^2 = 1$ may be parametrised by $x = \cos\theta$, $y = \sin\theta$.
(b) Either write the answer down by considering symmetry, or carry out the definite integral.

3574. Set the first derivative to zero for SPs. Use a double-angle formula to solve. Classify with the second derivative. To sketch, join the dots.

3575. Denote the motions A for anticlockwise, B for no motion and C for clockwise. List the successful outcomes and calculate their probabilities.

3576. (a) Use standard trig values.
(b) Use some more standard trig values.
(c) Set $x = \frac{1}{2}$ in the equation from part (b).

3577. Since the product is 12, the values may be $(1, 2, 6)$ or $(1, 3, 4)$ or $(2, 2, 3)$, in some order. This gives a possibility space of $6 + 6 + 3 = 15$ outcomes.

3578. (a) Use the quotient rule.
(b) You can take the limit $x \to -\infty$ immediately. To take the limit $x \to \infty$, firstly divide top and bottom by e^x.
(c) Find the y coordinates of the stationary points, and then join the dots!

3579. (a) Write $C = 180° - A - B$. Use the symmetry of the tan function to get rid of the 180°. Then expand with a compound-angle formula.
(b) Substitute the result from (a) into the LHS as an expression. Simplify this to reach the RHS.

3580. (a) Use the distributions
i. $M \sim N\big(60, 12^2\big)$,
ii. $\bar{M} \sim N\big(60, {}^{12^2}/_5\big)$.
(b) Consider the validity of the application of the normal distribution to such a group.

3581. Square both sides and simplify the trig with a double-angle formula. Then explain why some of these (x, y) points won't appear in the original graph.

3582. For the cubic to be invertible over $\mathbb{R}$, it must be one-to-one. So, it must have no turning points.

3583. (a) Find the equation of the tangents at $x = 0$ and $x = 1$. Alternatively, use one iteration of N-R.
(b) The iteration will be periodic.

3584. The sum of the interior angles is $(n-2)\pi$. Consider the mean of the AP.

3585. (a) Find the lift on the balloon by considering the situation before the monkey starts to climb. Then use this value after the monkey starts to climb to find accelerations.
(b) Find the acceleration of the monkey relative to the balloon, and use a single *suvat*.

3586. (a) Consider 3D Pythagoras.
(b) In the positive octant of (x, y, z) space, the equation is $x + y + z = 1$. Truncated to the first octant, what shape does this produce?

3587. Multiply top and bottom by e^x to eliminate the negative powers. Then factorise the top.

3588. Use the substitution $y = 2x + 3$. Then integrate by parts, considering $\ln y$ as $1 \cdot \ln y$ and setting $u = \ln y$.

3589. Statement S is true for one of these cases, but not the other.

3590. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. Find the total volume of water, then find an expression for the volume in the cone. Subtract these to find the volume in the sphere.

3591. Use a compound-angle formula, and then a double-angle formula. Then simplify the inlaid fractions by multiplying top and bottom by $(1 - \tan^2\theta)$.

3592. Work out the behaviour of the denominator. In particular, look at its symmetry (in the x axis) and its range.

3593. The hypothesis is true. Substitute the proposed solution $y = A\,\mathrm{f}(x)$ in and see what happens.

3594. (a) As two transformations, this is reflection in $y = x$ followed by reflection in the y axis.
  (b) Use the fact that the symmetry of $y = \mathrm{f}(x)$ is rotational around the origin.

3595. (a) Use the first Pythagorean trig identity to get rid of $\cos x$. Then factorise, explaining why the root $\sin x = 0$ is to be rejected.
  (b) Write $x \equiv 2 \cdot \frac{1}{2}x$ to use a double-angle formula. Then divide through by $\cos^2 \frac{1}{2}x$.
  (c) Use the second Pythagorean trig identity to produce a quadratic in $\tan \frac{1}{2}x$.

3596. Multiply out the right-hand sides.

3597. There are two possibilities, because the difference in position could be ± 1.

3598. This is false. Consider a linear function.

3599. (a) Find the derivative and substitute into the DE.
  (b) Find $\frac{dy}{dx}$ by the product rule. Then substitute into the DE and solve.

3600. (a) Consider harmonic form.
  (b) Try a few values on the calculator.

——— END OF 36TH HUNDRED ———